Bonus 1: Solution

(a) The precise definition of

$$\lim_{x \to a} f(x) = L$$

is: for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$ so that $|x - a| < \delta$ implies $|f(x) - L| < \epsilon$.

What this is saying is if one wants $f(x)$ to be close to L (within ϵ distance), all we have to do is make x close to a (within δ distance). So this formalizes the idea that "as x gets close to a, $f(x)$ gets close to L".

(b) Given ϵ, we need to find a δ such that $|x-2| < \delta$ implies $|(3x-2)-4| < \epsilon$.

But $|(3x - 2) - 4| = |3x - 6| = 3|x - 2|$. So to get $3|x - 2| < \epsilon$, we need $|x - 2| < \epsilon/3$. So if we choose $\delta = \epsilon/3$, then whenever $|x - 2| < \delta$ we would have

$$|f(x) - 4| = |3x - 6| = 3|x - 2| < 3(\epsilon/3) = \epsilon$$

Thus $|x - 2| < \delta$ implies $|f(x) - 4| < \epsilon$, so by the definition of limit given above,

$$\lim_{x \to 2} f(x) = 4.$$

Note: with this precise definition of limit, one can prove many of the things that we only stated in class, such as

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

and

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$